Extension of Ground-Based Testing for Large Space Structures

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A major concern for future large space structures is the ability to verify their dynamic characteristics by ground test. This paper presents the results of the multiple boundary conditions test approach that provides a complete ground test of a large structure. This ground test, in turn, supplies the data necessary to construct a test-verified final mathematical model. Theoretical studies indicate that this approach can provide a better final model than a ground test of the full-scale very flexible structure in a 1-*g* field.

Introduction

THIS paper shows, by theory, that a test-verified mathematical model of a large flexible structure can be validated using a series of multiple boundary condition tests (MBCT) to the accuracy of a good ground model test performed on the total structure. The proposed series of tests takes advantage of the flexibility of the large structures—a flexibility that makes the existing standard modal test procedures impossible to implement on the ground. The MBCT approach promises to overcome one of the largest impediments to successfully placing large flexible structures in space: the need for good ground test data to verify the system's structural dynamic model.

Historically, all new designs of aircraft and space structures have been partially verified by full-scale dynamic ground tests prior to initial flight. Even with the recent advances in computers and finite element codes, which allow solution of large complex structures, testing of structures whose dynamic characteristics are significant to mission success have been possible only at great expense. A recent example is the full-scale modal test of the Shuttle orbiter. A review of modal tests performed on large structures reveals that the tests have been of value in detecting errors or omissions in the analytical model. If the errors had not been detected, the mission would have been severely compromised. Although in some selected programs, such as the Shuttle, scale model testing has been of value in validating the mathematical model, ground testing of the full-scale structure has been performed and always proved to reveal additional valuable data.

For a number of years, the requirement for large flexible structures that must be precisely pointed and/or maintain an accurate geometric surface has been anticipated. In contrast to smaller, more rigid structures, these structures require verification of the system dynamic model to satisfy several factors: 1) the required knowledge of the modal characteristics is often more stringent; 2) the modal characteristics are more dense; 3) structural redundancy implies a large number of modes that have nearly identical eigenvalues; and 4) the inability to conduct a ground test on the full-scale structure because of its flexibility, the influence of the gravitational field, and other adverse environments on the ground.

The control of large structures in space will be dependent on the development of accurate analytical models that have been verified by test data at a few select operational configurations. The importance of verifying this mathematical model has led to the solar array flight experiment (SAFE). SAFE was a large unfurlable solar array flown on the Shuttle orbiter in the fall of 1984. Other structural dynamics flight experiments such as the Langley Research Center's MAST[1] program are planned for the future. A major objective in these flight tests is a more accurately developed mathematical model that will predict a large space structure's behavior in space.

Background

For a number of years, a large deployable wrap-rib antenna (WRA)[2,3] up to 150 m in diameter has been in development; a constraint was deployment in orbit from a single Shuttle launch. A decision was made to build and test a three-gore section of a full-scale 55 m diam antenna to help focus the technology requirements of large space structures. The WRA is illustrated in space and under test conditions in Figs. 1 and 2. In addition to demonstrating the reliability of its deployment, a major goal was to demonstrate the ability (or the limitation) to ground test the structure to verify its mathematical model.

The WRA is extremely light and flexible, to the extent that the test specimen was expected to collapse if deployed on Earth. As predicted, each of the graphite/epoxy composite ribs had to be supported by five near-equally spaced vertical cables to deploy the antenna section and prevent its collapse under the gravitational field.

Initially, the ability to ground test the ribs to verify the structural model appeared to be a very difficult challenge. The ribs were very flexible and often appeared to be excited by air currents in the building. Additionally, inspection of the ribs indicated small variations in the direction of the graphite fibers, thickness of the composite material, thickness of the adhesives, splices connecting segments of the ribs, and the overall cross-sectional geometry of the ribs. Small local differences could possibly accumulate and result in larger changes in the overall dynamic characteristics. Preliminary evaluations indicated the advantages of establishing the dynamic characteristics of each rib manufactured for flight. This would systematically place the higher frequency ribs in a pre-established pattern to obtain an antenna system with more favorable dynamic characteristics.

Faced with these challenges, the MBCT was conceived. To demonstrate the testability of the structure, a series of static and dynamic tests were performed on a single rib. Even though the limitations in schedule and funds dictated an unsophisticated series of tests, numerous tests on the rib with various boundary conditions were performed from which good test data were acquired.

MBCT Approach

Experience gained from WRA testing led directly to the MBCT approach for ground testing structures in this class.

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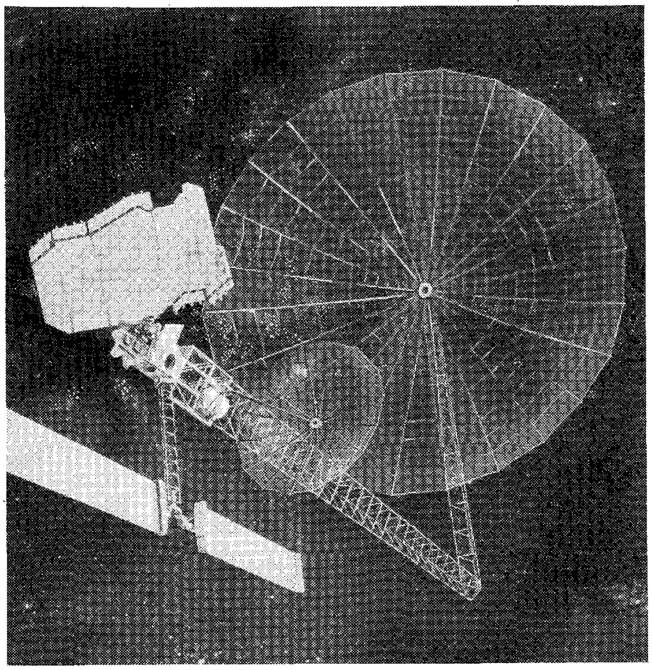

Fig. 1 Wrap-rib antenna deployed in space.

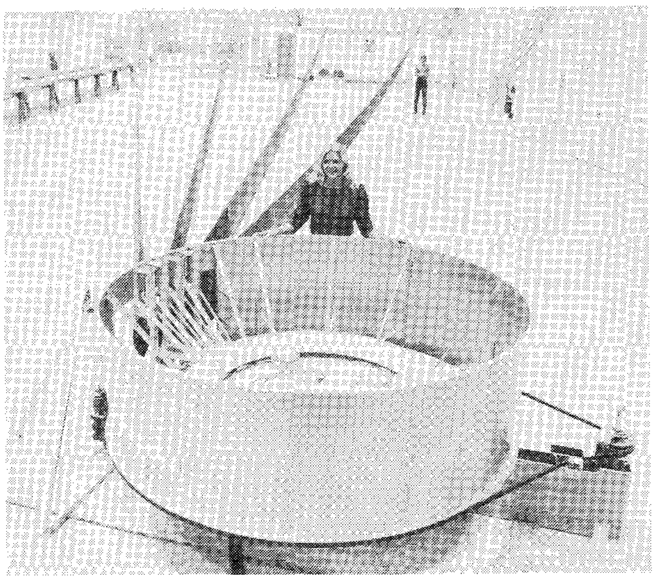

Fig. 2 Wrap-rib antenna deployed under test conditions.

These structures, as we have seen, must be supported with a constraint system to allow testing in the fully deployed geometry.

Because any variation in the modeling parameters in the vicinity of the supports will go undetected in a single test configuration, the system should be analyzed and tested in a variety of constraint conditions with the constraint locations varied during testing. This will provide the data necessary to upgrade the mass and stiffness parameters in the structural model. Additionally, the increased amount of test data will improve the parameter estimates used for the mathematical model. This is the technical reasoning behind the MBCT approach.

Because the constraint system will absorb significant energy from the structure, damping coefficients obtained by MBCT are not immediately applicable to a flight model. There is no one-to-one correspondence between the test and flight frequencies; thus, the measured damping from the test will provide typical damping coefficients for the structure only in the measured frequency range. Even for this application, the damping information obtained will be unusually suspect.

Theoretical Results

To compare the accuracy of the MBCT approach to the accuracy of a conventional modal test, it is necessary to describe mathematically the process by which test data are incorporated into a modal structural model. As with most formal analyses of this type, the nonlinear least-squares technique is used.[4] The equations from which analytical modal data are calculated are linearized and curve fitted to the test data to give revised estimates for the underlying mass and stiffness parameters. These revised estimates are then used to recalculate the modal data. This process is repeated until the revised mass and stiffness parameters no longer change appreciably or until the correlation between the mathematical and test results is acceptable.

The input data for a model (superscript 0) are represented by a mass matrix M^0 and stiffness matrix K^0. Also, the results of the Eigen transformation are represented by the system nautral frequencies f^0. Mode shapes and load coefficients are usually also available; however, to keep the analysis straightforward, only frequency data will be discussed here. Finally, modal test data (superscript t), represented by the test frequencies f^t, are available in an abbreviated form of the model data. The objective is to use the test data to modify the mass matrix and stiffness matrix such that the test and model data agree to the extent possible.

Observe that the frequencies extracted from the model are continuously dependent on the mass and stiffness parameters. The mass and stiffness are transformed through the Eigen transformation into frequencies. On an element-by-element basis (or group of elements representing a physical entity, such as area), the transformed quantities can be represented as Taylor series expansions in terms of the original mass and stiffness parameters, as

$$f(m_i, k_j) = f^0 + \left(\frac{\partial f}{\partial m_i}\right)(m_i - m_i^0) + \left(\frac{\partial f}{\partial k_j}\right)(k_j - k_j^0) + \ldots \tag{1}$$

The continuous quantities are represented by $f(m_i, k_j)$, m_i, and k_j, where the subscript means that the notations are to be repeated over all choices of i and j. Subscript i counts individual terms in the mass matrix and subscript j counts individual stiffnesses, which, when added together, form the stiffness matrix.

Where the model result f^0 is moved to the left side of Eq. (1) and higher-order terms are dropped, a local approximation in terms of differences between the model parameters and the continuous variables is obtained. This can readily be fit as described in the section on curve fitting. First, a table of differences between the test and model results must be prepared; next, the mass and stiffness terms must be identified; and, finally, the derivatives must be obtained as described in the section on the perturbation technique. The resulting equations can be represented in the following matrix form:

$$\{f^t - f^0\} = \left[\frac{\partial f}{\partial m_i} \middle| \frac{\partial f}{\partial k_j}\right] \left\{\frac{m^1 - m^0}{k^1 - k^0}\right\} \tag{2}$$

where m^1 and k^1 represent the revised mass and stiffness parameters after the first iteration. The process is repeated using the same test data, but at different values of the derivatives, and using the revised m^1 and k^1 values in place of m^0 and k^0 until the differences between the revised stiffness and mass terms are acceptably small. The Eigen transformation for the model must be recalculated at each step in the iteration.

Curve Fit Procedure

The least-squares curve fit proceeds by gathering up all the test and model data into the following system of equations:

$$\underset{n\times 1}{\{Y\}} = \underset{n\times nv}{[X]} \underset{nv\times 1}{\{B\}} + \underset{n\times 1}{\{E\}} \tag{3}$$

where Y is just a vector of the observed differences between the test and model results for the frequencies. The matrix X is composed entirely of first derivatives of the frequencies with respect to the mass and stiffness elements of the model. The vector B is calculated by the procedure and is composed of the difference between the new and old model values for the mass and stiffness elements. The terms $Y=XB$ are illustrated in Eq. (2). The constant n is the number of frequencies and the constant nv is the number of mass and stiffness parameters.

To discuss the accuracy to which the estimates of B are known, it is necessary to make some distributional assumption about the errors observed in curve fitting equation (2). Thus, the error vector E is introduced. It is considered to be normally distributed with mean of zero and variance equal to the matrix s^2I. The constant s^2 is calculated by the curve-fit procedure. The matrix I is just an identity matrix.

The least-squares solution for B and also the solution that minimizes s are given by

$$\underset{nv\times 1}{B} = \underset{nv\times n}{[X'} \ \underset{n\times nv}{X]^{-1}} \ \underset{nv\times n}{[X']} \ \underset{n\times 1}{Y} \tag{4}$$

To discuss the accuracy to which a model can be refined, it is necessary to estimate the tolerance of B (i.e., the square root of the variance of B). The variance of the estimator in Eq. (4) can be shown to be [5]

$$\underset{nv\times nv}{V(B)} = s^2 \ \underset{nv\times n}{[X'} \ \underset{n\times nv}{X]'^{-1}} \tag{5}$$

Equation (5) is a matrix (the covariance matrix); thus, only the square root of the diagonal terms (the variance) is used as a tolerance estimate. Finally, to estimate the variance using Eq. (5), it is necessary to have an estimate for s^2. First, one must define an error term as shown in Eq. (6); then the estimate for s^2 is as shown in Eq. (7):

$$\underset{n\times 1}{\{E\}} = \underset{n\times 1}{\{Y\}} - \underset{n\times nv}{[X]} \ \underset{nv\times 1}{\{B\}} \tag{6}$$

$$\underset{n\times 1}{s^2} = \underset{1\times n}{(E'} \ \underset{n\times 1}{E)} \ / \ (n-nv) \tag{7}$$

To prove that the MBCT procedure can refine a model to the same accuracy obtained from a conventional modal test, it is necessary to show that the variance in Eq. (5) can be made equally small. As n becomes large, the value for s^2 will approach some value indicating the amount of scatter in the data. It does not approach zero because the number of error terms in E also increases as n increases. The individual parameter tolerances will decrease as n increases. This is because the number of terms in X increases; thus, individual terms in $[X' \ X]$ (which has fixed dimensions $nv\times nv$) become larger and those in the inverse of $[X' \ X]$ become smaller. As n becomes very large, the standard error s^2 approaches the correct value and the tolerance on the coefficient estimates [in Eq. (5)] approaches zero. Using either the MBCT or conventional modal test, the data can be obtained to whatever accuracy is required through the acquisition of enough data.

There is really only one major qualification that needs to be made. The derivation is based on the assumption that the error is normally distributed with mean of zero. In practice, this may not be true. Nonlinearity in the modal response may tend to give a variety of estimates for B under varying boundary conditions. This situation is no different for the MBCT technique than for other modal techniques. Usually, a variety of stinger locations are used during modal testing; thus, there are often a variety of boundary conditions. In the past, this situation has been manageable and, hopefully, it will continue to be manageable with the MBCT technique. The only way to really know is to try it.

There are significant advantages to the conventional modal test. In particular, it is free of the constraint system and the errors that the system introduces (raising s^2). Also, in the conventional test, the boundary conditions are more flightlike; thus, the modeling errors uncovered during a conventional test tend to be more relevant. There are also significant advantages to the MBCT technique. Considerably more data can be acquired (raising n). Also, the constraint system helps identify where the modeling errors are physically located by isolating the mode shape errors between constraint locations.

In either case, the theoretical limits to the accuracy that might be achieved (e.g., the system thermal noise floor, knowledge of instrument locations, and limitations on the size of the computer model) are really quite small. Suppose the error from both types of testing is negligible. Under these conditions, all the errors in Eq. (6) would be zero, the estimated standard error in Eq. (7) would be zero, and the variance in Eq. (5) would be zero. Without introducing error into the simulation, it is possible to show that the MBCT is theoretically sound simply by demonstrating that the procedure outlined above will converge. Without errors, both a MBCT and a conventional model test have identical tolerance estimates (e.g., zero).

The Perturbation Technique

The partial derivatives in Eq. (2) can be evaluated in closed form using the perturbation method. Observe that both the original (e.g., m^0, k^0, f^0, and ϕ^0) and perturbed (e.g., m^1, k^1, f^1, and ϕ^1) quantities are the result of Eigen transformations; thus, orthogonality holds for both systems. There is a further assumption that the mode shapes form a basis in n space. This implies that a full set of modes must be calculated. After a great deal of manipulation, it is possible to show that the partial derivatives required in Eq. (2) are

$$\frac{\partial f_i^2}{\partial k_{rs}} = \frac{1}{4\pi^2} \frac{(\phi_{ri}\phi_{si})}{\{\phi_i'\} \ [M] \ \{\phi_i\}} \tag{8}$$

$$\frac{\partial f_i^2}{\partial m_{rs}} = -4\pi^2 f_i^2 \frac{\partial f_i^2}{\partial k_{rs}} \tag{9}$$

where f_i and $\{\phi_i\}$ are the frequency in Hertz and eigenvector of the ith mode, respectively, ϕ_{ri} the rth component of the ith eigenvector, m_{rs} the rth row and sth column element of the stiffness and mass matrix, respectively, and $[M]$ the mass matrix.

Simulation Examples

A simple beam simulation was run using NASTRAN to generate data that can be used to demonstrate the MBCT procedure. A simply supported beam was used as the conventional modal test model and a series of roller constraints were imposed simulating six individual test configurations for the MBCT model as illustrated in Fig. 3. Errors were introduced in the stiffness matrix and nine different combinations of the test model data were used in a nonlinear least-squares curve fit to find the errors.

The basic model was constructed of 16 two-dimensional beam elements with a variable spacing of either 1 or 2 ft over a 20 ft total span. The moment of inertia I_x of all elements was assumed to be 0.08333 in.^{4}. The mass was distributed uniformly at 8.986 × 10^{-4} slug/in.^{3}. Young's modulus was 30 × 10^6 psi and Poisson's ratio was 0.3. Under these conditions, the modal frequencies come out as shown in Table 1 for the conventional modal test model and as shown in Table 2 for each of the six constrained models.

Test results were simulated by introducing a 10% additional moment of inertia at element 4 and a 20% additional moment of inertia at element 10. To reduce the number of derivatives that need to be calculated, the perturbed models were assumed to be correct (i.e., the test results) and the unperturbed model was taken as the starting point for analysis (i.e., the uncorrected model). In this way, the starting point for each analysis (and thus the derivatives) is always the same.

Ten modes of the conventional modal test model were used in the nonlinear least-squares fit as follows:

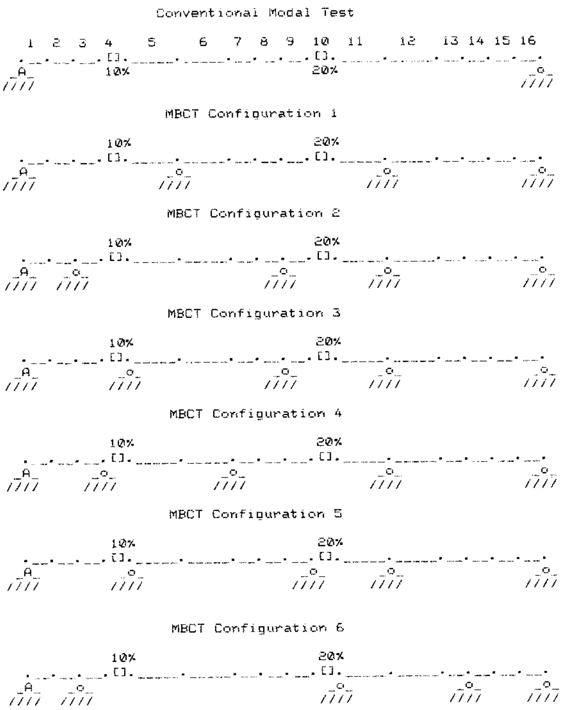

Fig. 3 Simply supported beam test configurations.

Table 1 Simulated model and test results for conventional modal test, Hz

Mode	Model	Test
1	1.44	1.45
2	5.83	5.86
3	12.70	12.82
4	22.85	23.06
5	35.88	35.97
6	51.66	52.24
7	73.75	74.08
8	82.45	83.13
9	109.10	110.59
10	174.72	175.29

1) Ten modes from the conventional modal test yielding 10 frequencies.

The resulting revised parameter estimates for the first two iterations of the conventional modal test model are shown in Table 3. As one would expect, the model parameters converge rapidly to the correct values using the conventional modal test technique. To compare this to the MBCT test, four combinations of test configurations from Fig. 3 were considered as follows:

2) Five modes each from configurations 1 and 2 yielding 10 frequencies.

3) Four modes each from configurations 1 and 2 yielding eight frequencies.

4) Three modes each from configurations 1 and 2 yielding six frequencies.

5) Two modes each from configurations 1 and 2 yielding four frequencies.

The resulting revised parameter estimates for the first two iterations are also shown in Table 3. Comparison of the results from the conventional test model and the MBCT model shows that the MBCT model actually converges slightly faster than the conventional test model. This demonstrates that the MBCT technique is fundamentally sound and can be used to validate a modal structural model to the same or higher degree of accuracy achieved historically.

To further investigate any limitations there might be to combining modes from a MBCT test, four additional combinations of test configurations were considered:

6) Two modes each from configurations 2-6 yielding 10 frequencies.

7) Two modes each from configurations 2-5 yielding eight frequencies.

8) One mode each from configurations 1-6 yielding six frequencies.

9) One mode each from configurations 1-4 yielding four frequencies.

Table 2 Simulated model and test results for MBCT, Hz

	Configuration					
Mode	1	2	3	4	5	6
			Model			
1	11.62	15.52	19.13	17.01	18.19	10.15
2	19.06	19.83	24.08	25.39	20.83	27.43
3	22.67	41.97	44.65	37.93	42.43	37.81
4	42.73	57.14	53.94	69.90	58.46	61.56
5	71.05	73.65	72.21	75.29	71.01	75.73
			Test			
1	11.73	15.61	19.20	17.17	18.42	10.21
2	19.19	19.85	24.27	25.58	20.92	27.49
3	22.75	42.58	44.99	38.17	42.61	38.33
4	44.48	58.04	54.84	70.05	59.02	61.83
5	71.32	73.91	72.34	75.91	71.08	75.90

Table 3 Results of estimated parameters, iterations 1 and 2, ΔI_4 and ΔI_{10} [theoretical values ($\Delta I_4 = 0.00834$, $\Delta I_{10} = 0.01667$)]

Case	Iteration 1		Iteration 2		Configuration
1) ΔI_4	0.005884	70%	0.008121	97.38%	Conventional modal test,
ΔI_{10}	0.015499	93%	0.016722	100.31%	10 frequencies total
2) ΔI_4	0.007084	85%	0.008330	99.88%	MBCT configuration 1-2,
ΔI_{10}	0.015291	92%	0.016640	99.82%	10 frequencies total
3) ΔI_4	0.007849	94%	0.008339	99.99%	MBCT configuration 1-2,
ΔI_{10}	0.014891	89%	0.016636	99.80%	8 frequencies total
4) ΔI_4	0.007716	93%	0.008338	99.98%	MBCT configuration 1-2,
ΔI_{10}	0.014683	88%	0.016630	99.76%	6 frequencies total
5) ΔI_4	0.007544	91%	0.008336	99.95%	MBCT configuration 1-2,
ΔI_{10}	0.014360	86%	0.016625	99.73%	4 frequencies total

Table 4 Results of estimated parameters, iteration 1, ΔI_4 and ΔI_{10} [theoretical values ($\Delta I_4 = 0.00834$, $\Delta I_{10} = 0.01667$)]

Case	Iteration 1		Configuration
6) ΔI_4	0.002424	30%	MBCT configuration 2-6,
ΔI_{10}	0.016615	100%	10 frequencies total
7) ΔI_4	0.002248	27%	MBCT configuration 2-5,
ΔI_{10}	0.016620	100%	8 frequencies total
8) ΔI_4	0.007891	95%	MBCT configuration 1-6,
ΔI_{10}	0.014175	85%	6 frequencies total
9) ΔI_4	0.007833	94%	MBCT configuration 1-4,
ΔI_{10}	0.014170	85%	4 frequencies total

The resulting revised parameter estimates for the first iteration are shown in Table 4. As can be seen, the process converges rapidly for all cases with the exception of parameter ΔI_4 in cases 5 and 6. Inspection of the test configurations used in cases 6 and 7 (see Fig. 3) shows that a large number of the test configurations used in cases 6 and 7 have constraints adjacent to element 4. This illustrates that the constraint configurations must be considered carefully when using the MBCT technique.

Conclusions

While full-scale ground testing of large deployable space structures is going to be difficult, it is required in some combination with flight testing. There are enough unknowns in the manufacturing process for this class of structures that full-scale testing will be required to have a high degree of confidence in the models.

The major problem with full-scale ground testing is the extreme flexibility of the structure. This necessitates a system of constraints during ground testing. However, through multiple boundary condition testing (MBCT), it is possible to effectively eliminate the influence of the constraints, allowing the model to be improved to the same or greater degree of accuracy as that obtained from a conventional modal test.

The authors have demonstrated mathematically that the MBCT concept can be used to develop a mathematical model that can be verified by ground tests. The MBCT provides an approach that may overcome one of the greatest obstacles in the use of large structures in space, i.e., the ability to ground test the structure. This paper presents only the first in many studies necessary to validate the concept. The ultimate goal is to apply the MBCT method to validate a mathematical model of a structure that is actually flown and tested in space, thus providing the ultimate verification of the proposed concept.

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